

AN INTRODUCTION TO SPECIAL STRUCTURES ON COMPLEX MANIFOLDS

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ABSTRACT

The first part of the course will provide an introduction to complex and almost complex geometry. There will be a concise overview of the theory of holomorphic functions of several complex variables, omitting analytical details, leading to basic examples of complex and almost complex manifolds. We will also discuss the Dolbeault, Bott-Chern and Aeppli cohomologies. The second part of the course will focus on the Hermitian geometry, starting with the Kaehler manifold and describing their cohomological properties. Then, the existence of special Hermitian metrics on compact complex manifolds without Kaehler structure will be investigated. Explicit examples and computational techniques of cohomological invariants will be carefully described.

Notes for the course will be provided.

1. COMPLEX GEOMETRY.

1.1 Introduction to the theory of holomorphic functions of several complex variables.

1.2 Complex structures. Complex projective spaces. Complex tori.

1.3 Almost complex structures. Newlander-Nirenberg theorem.

1.4 (p,q) -forms on complex manifolds. $\bar{\partial}$ -operator. Dolbeault complex. Aeppli and Bott-Chern cohomology.

2. HERMITIAN MANIFOLDS AND SPECIAL STRUCTURES.

2.1 Hermitian and Kaehler metrics. Kaehler metrics in local coordinates. Examples of Kaehler manifolds.

2.2 Cohomological properties of compact Kaehler manifolds. The $\partial\bar{\partial}$ -Lemma.

2.3 Chern and Bismut connections.

2.4 Strong Kaehler with torsion, Astheno Kaehler, balanced metrics.

2.5 p -Kaehler structures. Families of p -Kaehler structures. Cohomological properties of p -Kaehler manifolds.

REFERENCES

[1] D. Huybrechts, Complex Geometry: An Introduction, Springer Universitext, 2014.

[2] S. Kobayashi, Differential Geometry of Complex Vector Bundles, Princeton Legacy Library, 1987.

[3] J. Morrow, K. Kodaira, Complex manifolds. Reprint of the 1971 edition AMS Chelsea Publishing, Providence, RI, 2006. x+194

Notes for the course.