PhD Course

Duality Theory and Applications for Markov Processes

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Course overview

This course introduces the concept of duality for Markov processes and its applications in probability theory and statistical mechanics. Starting from basic definitions and classical examples, the course develops duality techniques for interacting particle systems and explores their role in non-equilibrium phenomena and hydrodynamic limits. Emphasis is placed on probabilistic intuition, explicit computations, and connections with algebraic structures.

Dates and schedule

- 16 September 2026 (10:00–12:00)
- 16 September 2026 (14:00–16:00)
- 18 September 2026 (10:00–12:00)
- 18 September 2026 (14:00–16:00)

Total 16 hours = 8 hours (4 lectures, 2 hours each) + 8 hours seminars

Prerequisites

Basic probability theory (Markov chains, stochastic processes), measure theory, and elementary functional analysis. Some familiarity with stochastic calculus and interacting particle systems is helpful but not required.

Plan of the lectures

Lecture 1 — Introduction to duality for Markov processes

- Definition of duality and basic properties
- Duality functions and interpretation
- Introductory examples:
  - Reflected Brownian motion and absorbed Brownian motion at the origin
  - Wright–Fisher diffusion and Kingman coalescent
- Basics of the Lie algebraic approach
- Markov chains with finite state space

Lecture 2 — Duality for interacting particle systems

- Independent random walkers
- Exclusion process and inclusion process
- Brownian energy process
- Duality in interacting systems
- Classical models:
  - Aldous averaging model
  - Kac's model
  - Kipnis–Marchioro–Presutti model

Lecture 3 — Non-equilibrium steady states

- Stationary non-equilibrium systems
- Dual absorbed processes
- Computation of stationary profiles via duality
- Closed-form solutions:
  - Independent random walkers
  - Harmonic process
  - Integrable heat conduction models

Lecture 4 — Applications to hydrodynamics

- Hydrodynamic limits via duality
- Functional law of large numbers
- Application to independent random walkers
- Macroscopic behavior from microscopic dual processes

Bibliography

- A. De Masi, E. Presutti, *Mathematical Methods for Hydrodynamic Limits*, Springer (1991)
- T. M. Liggett, *Interacting Particle Systems*, Springer (2005)
- C. Giardinà, F. Redig, *Duality for Markov processes: a Lie agebraic approach*, Springer (2025)
- Video lectures: Seguire in link "Video for minicourses" della pagina web `hhttps://ims.nus.edu.sg/events/intertwining-between-probability-analysis-and-statistical-physics/`