

PhD Course

Kinetic Equations: Classical Foundations and Modern Computational Perspectives

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Course overview

This course introduces kinetic equations as a bridge between microscopic particle/agent models and macroscopic continuum descriptions. The backbone is the classical micro–meso–macro viewpoint (from particles to kinetic PDEs and hydrodynamic/diffusive limits), with emphasis on modern numerical methodology (particle methods, grid-based discretizations, structure-preserving and asymptotic-preserving ideas). Two contemporary perspectives are included: kinetic models for collective dynamics in social sciences and links to modern ML developments (operator learning/surrogates for solution operators).

Format and duration

Total: 28 hours = 18 hours (6 lectures 3 hrs each) + 10 hours seminars (inside the course).

Prerequisites

Basic PDEs (transport/diffusion), functional analysis at an introductory level, and basic probability (helpful but not mandatory). Prior exposure to numerical methods for ODE/PDE methods is recommended.

Plan of the lectures

Lecture 1 — From micro to kinetic: models and scalings

- Particle/agent systems: interactions, noise, collisions; mean-field intuition.
- BBGKY / propagation of chaos (conceptual level).
- Canonical kinetic models: Boltzmann/BGK, Vlasov, Fokker–Planck.

Lecture 2 — Essential classical theory: structure and qualitative behavior

- Invariants and a priori bounds (mass, momentum, energy, depending on model).
- Entropy and dissipation (Boltzmann/BGK/Fokker–Planck); trend to equilibrium (overview).
- Homogeneous vs inhomogeneous settings; collisionless vs collisional regimes.

Lecture 3 — Particle-based numerics: Monte Carlo, DSMC, PIC

- Monte Carlo basics: bias/variance, sampling error, stabilization.
- DSMC ideas for collisional kinetics; accuracy considerations.
- PIC for Vlasov: deposition/field solve/push; noise control and conservation issues.

Lecture 4 — Grid-based and micro–macro numerics

- Grid discretizations: semi-Lagrangian / DG / discrete velocity (overview).
- Fast and structure-preserving spectral methods for collisional operators.
- Asymptotic-preserving (AP) philosophy and IMEX methods.

Lecture 5 — Beyond the classical paradigm: social and collective dynamics

- Wealth exchange models, opinion formation, swarming, traffic flows: agent-based models and kinetic/Fokker–Planck-type limits;
- Networks and structured contacts; kinetic/mean-field descriptions in epidemiology.
- Control and learning from data: optimal control, calibration and identification of interaction parameters.

Lecture 6 — Connections to modern ML and UQ

- Surrogate models, uncertainty quantification (UQ) and multi-fidelity methods.
- Physics informed machine learning: learning solution maps for Boltzmann and Landau equations
- Future directions and perspectives.

Assessment (optional)

1 short numerical mini-assignment *or* a 20-minute presentation on a recent paper chosen with the instructor.

References

- [1] C. Cercignani, R. Illner, M. Pulvirenti, *The Mathematical Theory of Dilute Gases*, Applied Mathematical Sciences 106, Springer, 1994.
- [2] C. Villani, *A Review of Mathematical Topics in Collisional Kinetic Theory*, in: S. Friedlander, D. Serre (eds.), *Handbook of Mathematical Fluid Dynamics*, Vol. I, North-Holland, 2002, pp. 71–305.
- [3] G. Dimarco, L. Pareschi, *Numerical methods for kinetic equations*, Acta Numerica **23** (2014), 369–520.
- [4] L. Pareschi, G. Toscani, *Interacting Multiagent Systems: Kinetic Equations and Monte Carlo Methods*, Oxford University Press, 2013.
- [5] S. Boscarino, L. Pareschi, G. Russo, *Implicit-explicit methods for evolutionary PDEs*, SIAM Mathematical Modeling and Computation, 2024.
- [6] Giacomo Albi, Lorenzo Pareschi, Giuseppe Toscani, Mattia Zanella, *Recent advances in opinion modeling: control and social influence*, Active particles. Vol. 1. Advances in theory, models, and applications, 49–98, Model. Simul. Sci. Eng. Technol., Birkhäuser/Springer, Cham, 2017.
- [7] Giacomo Dimarco, Liu Liu, Lorenzo Pareschi, Xueyu Zhu, *Multi-fidelity methods for uncertainty propagation in kinetic equations*, Panoramas & Synthèses 65, 49-124, Société Mathématique de France, 2025.
- [8] Wei Chen, Giacomo Dimarco, Lorenzo Pareschi, *Structure and asymptotic preserving deep neural surrogates for uncertainty quantification in multiscale kinetic equations*, to appear in SIAM MMS. Preprint arXiv:2506.10636, 2025.